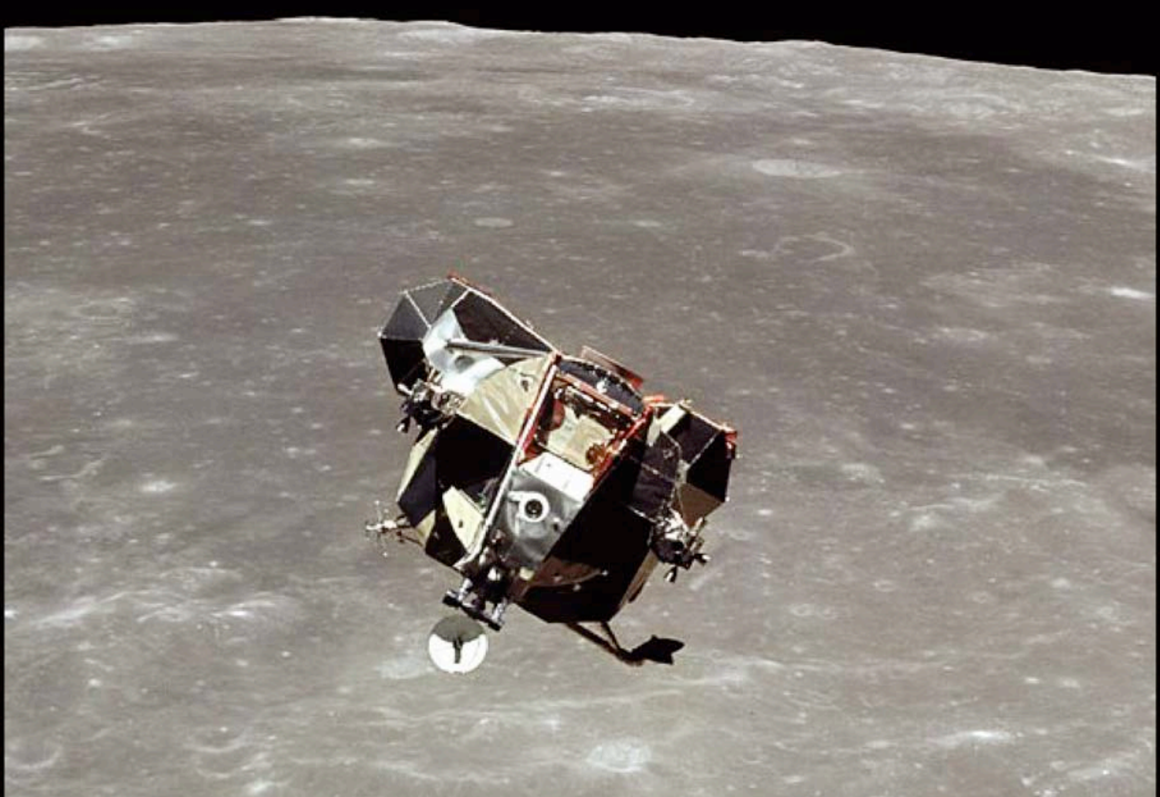
National Aeronautics and Space Administration



Lunar Math

How to Use this Book

Teachers continue to look for ways to make math meaningful by providing students with problems and examples demonstrating its applications in everyday life. Space Mathematics offers math applications through one of the strongest motivators- Space. Technology makes it possible for students to *experience* the value of math, instead of just reading about it. Technology is essential to mathematics and science for such purposes as “access to outer space and other remote locations, sample collection and treatment, measurement, data collection and storage, computation, and communication of information.” 3A/M2 authentic assessment tools and examples. The NCTM standards include the statement that "Similarity also can be related to such real-world contexts as photographs, models, projections of pictures" which can be an excellent application for Lunar data.

Lunar Math is designed to be used as a supplement for teaching mathematical topics. The problems can be used to enhance understanding of the mathematical concept, or as a good assessment of student mastery.

An integrated classroom technique provides a challenge in math and science classrooms, through a more intricate method for using **Lunar Math**. Read the scenario that follows

Mr. Smith and Ms. Green decided to team teach. Mr. Smith is teaching students about the solar system and the history of the Apollo Missions, looking toward the future of the next landings on the moon, in 2017. Ms. Green is teaching several math levels, with the same students that Mr. Smith teaches science. They decide to use the Lunar Math supplement to learn about the science and how the math applications will provide information about the moon. Ms. Green checks the *Alignment to Mathematical Standards*, in the front of Lunar Math, to determine which topics she can use with her 5 classes. All classes can use a review on scale drawings, probability and beginning geometry found in activities 1-5. This is where they begin. Mr. Smith has the students’ look at images on the internet of the moon, they begin with images provided by the Lunar and Planetary Institute and build a knowledge base about the surface of the moon. Ms. Green uses the first 5 activities from Lunar Math as students learn about the size of craters that impact the moon’s unprotected surface.

Lunar Math can be used as a classroom challenge activity, assessment tool, enrichment activity or in a more dynamic method as is explained in the above scenario. It is completely up to the teacher, their preference and allotted time. What it does provide, regardless of how it is used in the classroom, is the need to be proficient in math. It is needed especially in our world of advancing technology and space discovery.

This collection of activities is based on a weekly series of space science problems distributed to thousands of teachers during the 2004-2008 school years. They were intended for students looking for additional challenges in the math and physical science curriculum in grades 6 through 12. The problems are designed to be 'one-pagers' consisting of a Student Page, and Teacher's Answer Key. This compact form was deemed very popular by participating teachers.

The topic for this collection is **Lunar Exploration** and our upcoming return to the Moon in the next decade. It starts with NASA's launch in 2009 of the Lunar Reconnaissance Orbiter. This satellite will survey the Moon for water ice, and measure the Moon's radiation environment in preparation for manned landings between 2017 and 2020. For more information, visit the LRO website at

http://lunar.gsfc.nasa.gov/index.html

This booklet was created by the
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For more weekly classroom activities about astronomy and space science, visit

http://spacemath.gsfc.nasa.gov

Add your email address to our mailing list by contacting Dr. Sten Odenwald at

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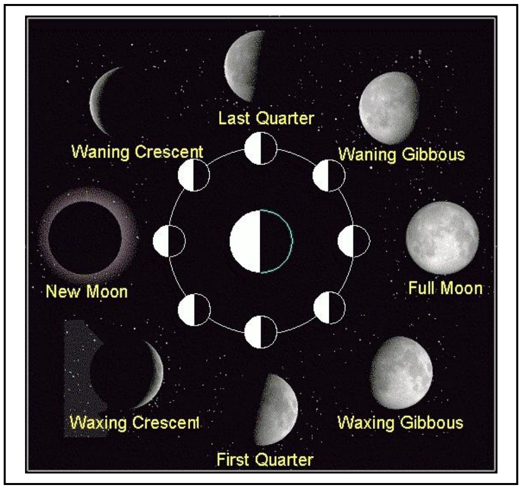
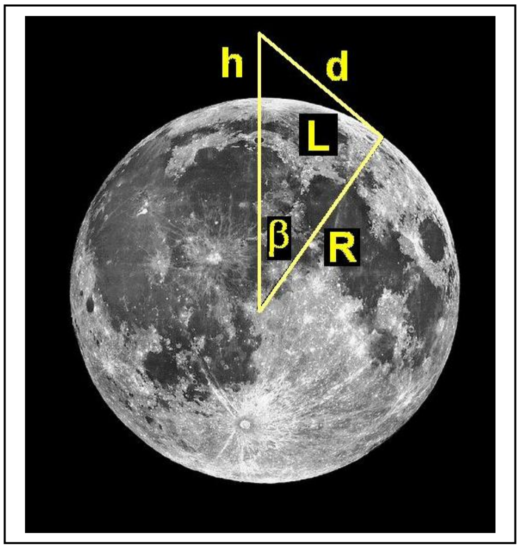
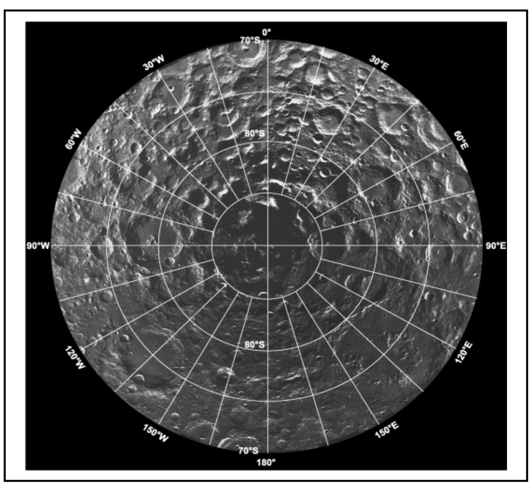
Introducing the Moon







The Moon is truly one of those celestial bodies that needs no introduction! We are all experts in observing it, noting its changing phases, and understanding its 'moonthly' motions in the sky!

As a topic for mathematical study, however, it offers many opportunities to combine both very simple, and very advanced, mathematics topics to further probe its many mysteries. For thousands of years, simple addition, subtraction, multiplication and division was all that was needed to master its chronology in the sky. The advent of telescopes in the 1600's, and the Space Age in the 1960's, however, opened up many new ways to learn about it as a mathematical object.

This booklet is part of a growing collection of mathematics problems for K12 students that explore many different aspects of the moon, from its craters to its thin atmosphere; from its warm interior to its outermost limits in space.

In 2009, NASA will launch the Lunar Reconnaissance Orbiter to photograph the lunar surface. It follows a growing international armada of spacecraft that are taking a closer look at the moon in preparation for humans to eventually return and set-up housekeeping. This will require using all of the available lunar resources, which Apollo astronauts discovered and catalogued between 1969-1972 during their historic visits. The search for water ice, and its discovery, would save us billions of dollars in having to drag our own water from Earth to support colonists. The lunar rocks will be chemically processed to extract the ingredients for 'home made' lunar rocket fuel for return journey to Earth, or even outbound trips to Mars!

This booklet will be your guide to some of the mathematical aspects of studying the moon, and eventually sending humans there to live and work.

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Cover and back page art credits: Front: Colony on moon (NASA); Solar eclipse (Fred Espenak); Apollo-11 LEM and earthrise (NASA). Back: The Lunar Rover (NASA Apollo 15).

Topics and Alignment with Mathematics Standards

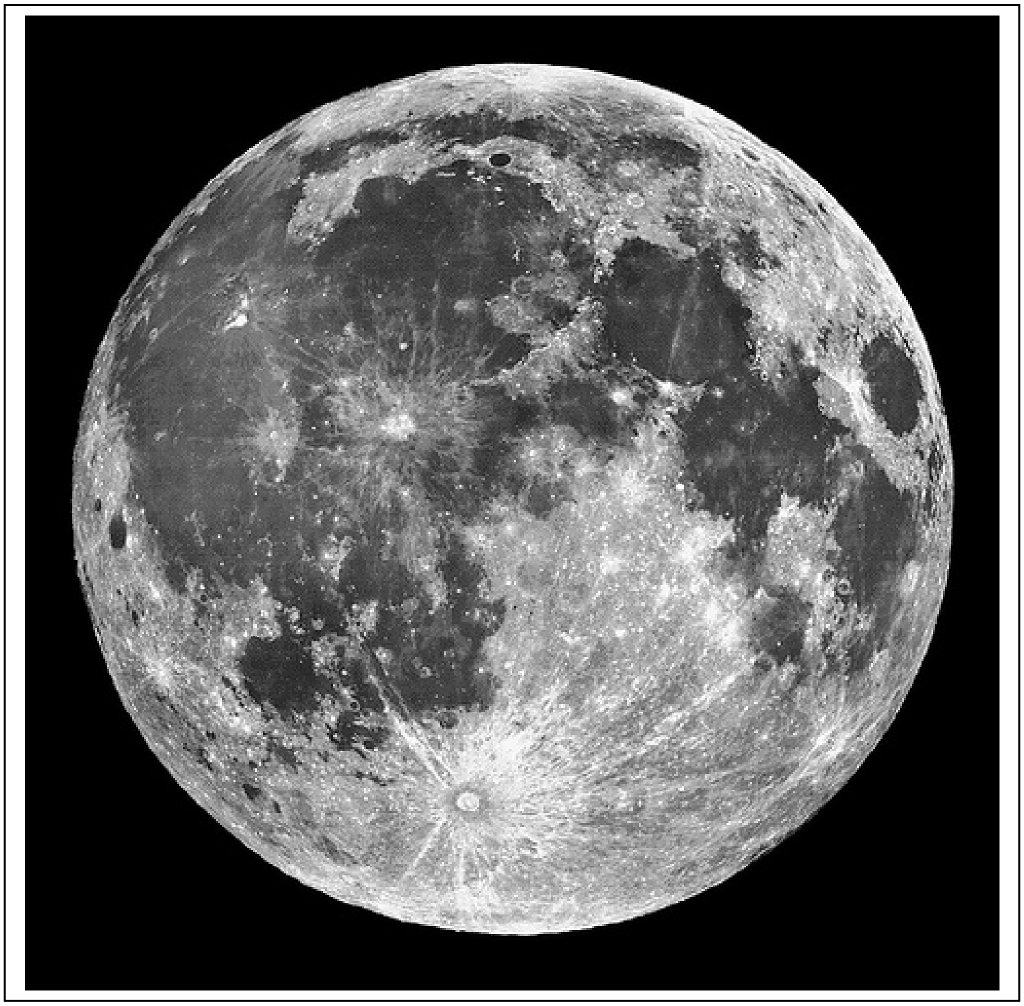
Topic	Problem Number																
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
Number Patterns																	
Area					x									x	x		
Volumes							x					x	x				
Probability				x										x			
Fractions				x											x		x
Decimals	x	x	x		x	x		x								x	x
Scale drawings	x	x	x												x	x	
Polygonal Areas					x												
Geometry					x	x		x	x	x							
Scientific Notation					x		x		x		x	x	x	x			
Unit Conversions	x	x	x			x	x	x			x	x	x	x	x	x	x
Pythagorean Theorem									x	x						x	
Sin, Cos, Tan									x	x							
Solving for X									x	x	x		x				x
Multivariable algebra											x		x				
Function Differentiation										x							

Applicable Standards (AAAS Project:2061 Benchmarks).

(3-5) - Quantities and shapes can be used to describe objects and events in the world around us. 2C/E1 --- Mathematics is the study of quantity and shape and is useful for describing events and solving practical problems. 2A/E1 **[Relevant problems 1,2,3]**

(6-8) Mathematicians often represent things with abstract ideas, such as numbers or perfectly straight lines, and then work with those ideas alone. The "things" from which they abstract can be ideas themselves; for example, a proposition about "all equal-sided triangles" or "all odd numbers". 2C/M1 **[Relevant problems 4,5,6,8,9,14,15,16]**

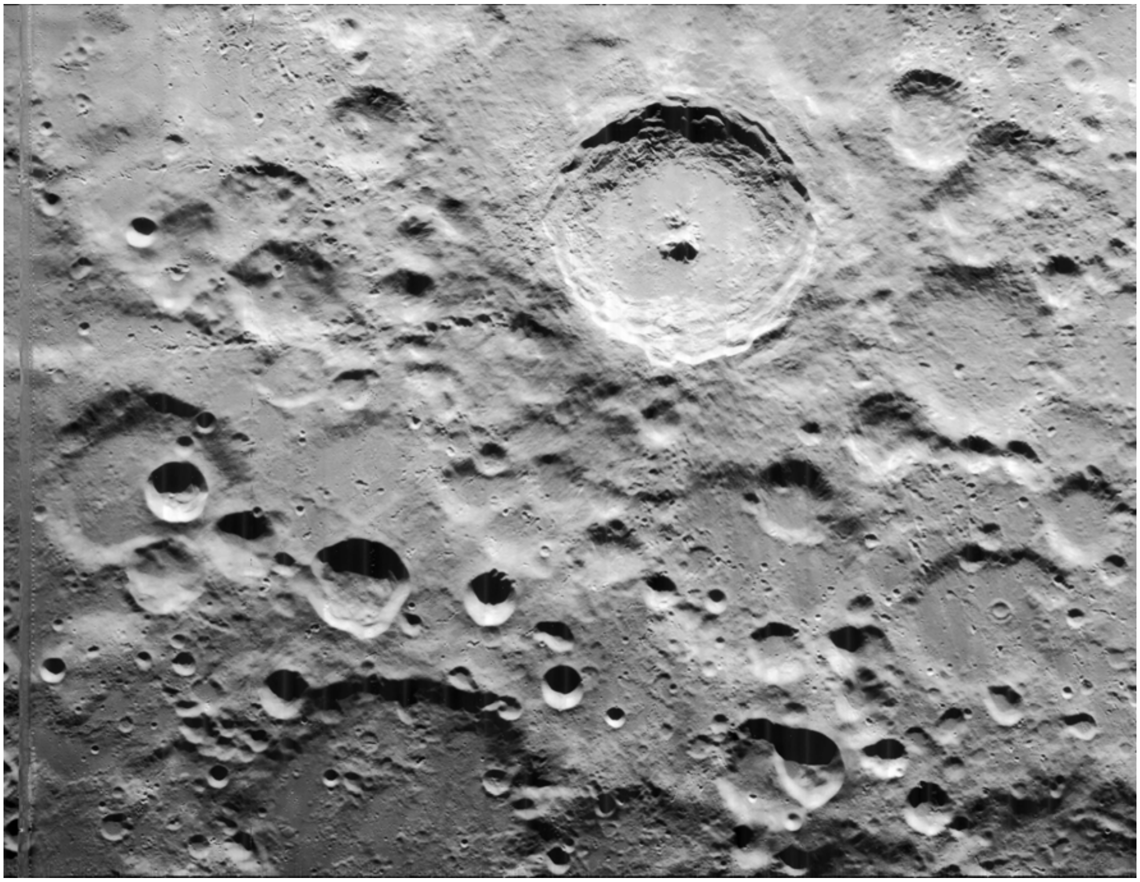
(9-12) - Mathematical modeling aids in technological design by simulating how a proposed system might behave. 2B/H1 ---- Mathematics provides a precise language to describe objects and events and the relationships among them. In addition, mathematics provides tools for solving problems, analyzing data, and making logical arguments. 2B/H3 ----- Much of the work of mathematicians involves a modeling cycle, consisting of three steps: (1) using abstractions to represent things or ideas, (2) manipulating the abstractions according to some logical rules, and (3) checking how well the results match the original things or ideas. The actual thinking need not follow this order. 2C/H2 **[Relevant problems 5,7,8,9,10,11,12,13,16,17]**



It is worth reminding students that humans have landed and walked on the moon. This happened during the US Apollo Program between 1969-1972.

Neil Armstrong	Apollo 11	July 21, 1969
Buzz Aldrin	Apollo 11	July 21, 1969
Pete Conrad	Apollo 12	November 19-20, 1969
Alan Bean	Apollo 12	November 19-20, 1969
Alan Shepard	Apollo14	February 5-6, 1971
Edgar Mitchell	Apollo 14	February 5-6, 1971
David Scott	Apollo 15	July 31 - August 2, 1971
James Irwin	Apollo 15	July 31 - August 2, 1971
John Young	Apollo 16	April 21 - 23, 1972
Charles Duke	Apollo 16	April 21 - 23, 1972
Eugene Cernan	Apollo 17	December 11-14, 1972
Harrison Schmidt	Apollo 17	December 11-14, 1972

Craters on the Moon



This is a NASA image taken by the Lunar Orbiter IV spacecraft as it captured close-up images of the lunar surface in May, 1967. The large crater at the top-center is Tycho. Other images from the Lunar Orbiter spacecrafts can be found at the Lunar Orbiter Photo Gallery (http://www.lpi.usra.edu/resources/lunarorbiter/) The satellite was at an altitude of 3,000 kilometers when it took this image, which measures 350 km x 270 km.

The scale of an image is found by measuring with a ruler the distance between two points on the image whose separation in physical units you know. In this case, we are told the field of view of the image is 250 kilometers x 270 kilometers.

Step 1: Measure the width of the lunar image with a metric ruler. How many millimeters long is the image?

Step 2: Read the explanation for the image and note any physical scale information provided. The information in the introduction says that the image is 350 kilometers along its largest dimension.

Step 3: Divide your answer to Step 2 by your answer to Step 1 to get the image scale in kilometers per millimeter to two significant figures.

Once you know the image scale, you can measure the size of any feature in the image in units of millimeters. Then multiply it by the image scale from Step 3 to get the actual size of the feature in kilometers to two significant figures.

Question 1: What is the diameter of the crater Tycho in kilometers?
Question 2: How large is the smallest feature you can see?
Question 3: How large are some of the smaller hills at the floor of the crater, in meters?
Question 4: About how large are the most common craters in the field?
Question 5: Which crater is about the same size as Denver, which has a diameter of about 25 km?

Answer Key:

Step 1: Measure the width of the lunar image with a metric ruler. How many millimeters long is the image?
Answer: 150 millimeters.

Step 2: Read the explanation for the image and note any physical scale information provided. The information in the introduction says that the image is 350 kilometers along its largest dimension.
Answer: 350 kilometers.

Step 3: Divide your answer to Step 2 by your answer to Step 1 to get the image scale in kilometers per millimeter to two significant figures.
Answer: 350 kilometers / 150 millimeters = 2.3 kilometers / millimeter.

Once you know the image scale, you can measure the size of any feature in the image in units of millimeters. Then multiply it by the image scale from Step 3 to get the actual size of the feature in kilometers to two significant figures.

Question 1: What is the diameter of the crater Tycho in kilometers?
Answer: About 35 millimeters x 2.3 km/mm = 80.5 kilometers in diameter which is 80 kilometers to two significant figures.

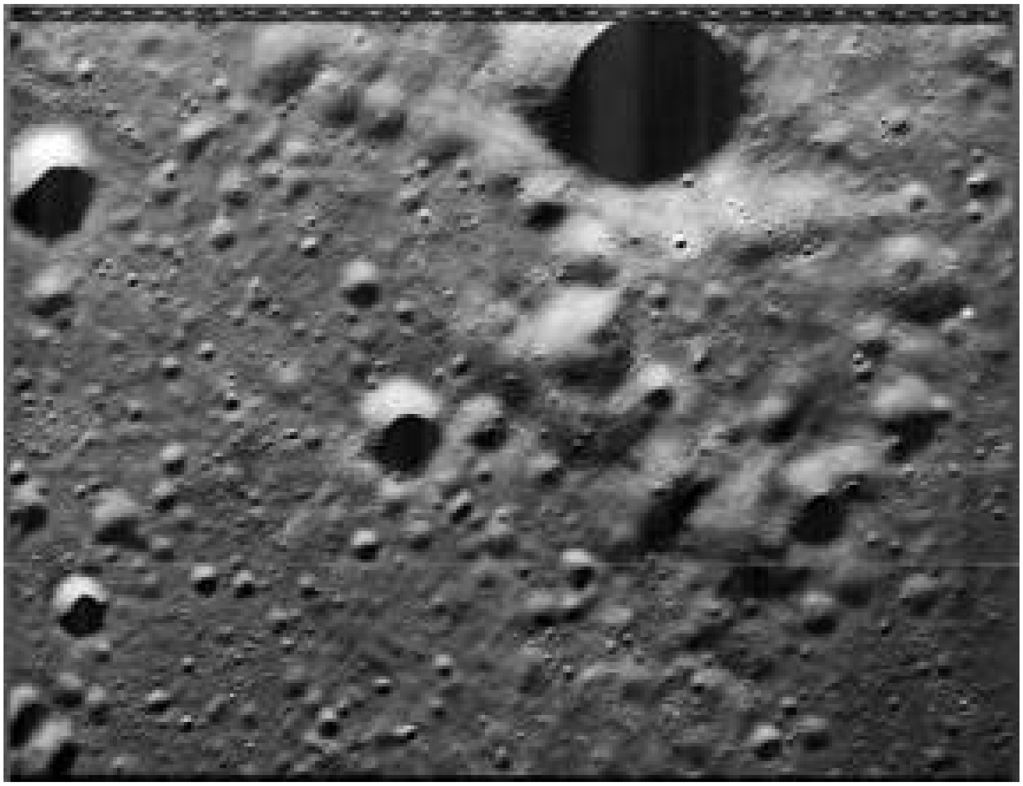
Question 2: How large is the smallest feature you can see?
Answer: There are many small details in the image, pits, hills, etc, that students can estimate 0.1 to 0.3 millimeters for a physical size of 0.2 to 0.7 kilometers since the measurement is only good to one significant figure.

Question 3: How large are some of the smaller hills at the floor of the crater, in meters?
Answer: These small features are about 0.1 millimeters across or 200 meters in size.

Question 4: About how large are the most common craters in the field?
Answer: The answer may vary a bit, but the small craters that are 0.5 millimeters across are the most common. These have a physical size of about 1 kilometer.

Question 5: Which crater is about the same size as Denver, which has a diameter of about 25 km?
Answer: In order to fit Denver into one of these lunar craters, it will have to appear to be about 25 km x (1.0 millimeter/2.3 km) = 11 millimeters across. There are three caters just to the right of Tycho that are about this big. Students should not get 'lost' trying to exactly match up their estimate with a precise lunar feature. 'Close-enough' estimates are good enough! See below comparison as a guide.

Craters on the Moon



This is a high resolution image of the lunar surface taken by NASA's Lunar Orbiter III spacecraft in February 1967 as it orbited at an altitude of 46 kilometers. It is located near the lunar equator. The field of view is 16.6 kilometers x 4.1 kilometers. Additional Orbiter images can be found at the Lunar Orbiter Gallery (http:// www.lpi.usra.edu/resources/lunarorbiter/). Because of the low sun angle, craters look like circles that are half-black, half-white inside!

The scale of an image is found by measuring with a ruler the distance between two points on the image whose separation in physical units you know. In this case, we are told the field of view of the image is 16.6 kilometers x 4.1 kilometers.

Step 1: Measure the width of the lunar image with a metric ruler. How many millimeters long is the image?

Step 2: The information in the introduction says that the image is 16.6 kilometers long. Convert this number into meters.

Step 3: Divide your answer to Step 2 by your answer to Step 1 to get the image scale in meters per millimeter to the nearest significant figure.

Once you know the image scale, you can measure the size of any feature in the image in units of millimeters. Then multiply it by the image scale from Step 3 to get the actual size of the feature in meters to the nearest significant figure.

Question 1: How big is the largest crater in the image?
Question 2: How big is the smallest crater in the image, in meters?
Question 3: About what is the typical distance between craters in the image?
Question 4: How far would you have to walk between the largest, and next-largest craters?

Answer Key:

The scale of an image is found by measuring with a ruler the distance between two points on the image whose separation in physical units you know. In this case, we are told the field of view of the image is 16.6 kilometers x 4.1 kilometers.

Step 1: Measure the width of the lunar image with a metric ruler. How many millimeters long is the image?
Answer: 134 millimeters.

Step 2: The information in the introduction says that the image is 16.6 kilometers long. Convert this number into meters.
Answer: 16600 meters.

Step 3: Divide your answer to Step 2 by your answer to Step 1 to get the image scale in meters per millimeter to the nearest significant figure.
Answer: 16600 meters / 134 millimeters = 124 meters / millimeter.

Once you know the image scale, you can measure the size of any feature in the image in units of millimeters. Then multiply it by the image scale from Step 3 to get the actual size of the feature in meters to the nearest significant figure.

Question 1: How big is the largest crater in the image?
Answer: The one at the top is about 25 millimeters across. 25 mm x 124 meters/mm = 3,100 meters or 3.1 kilometers.

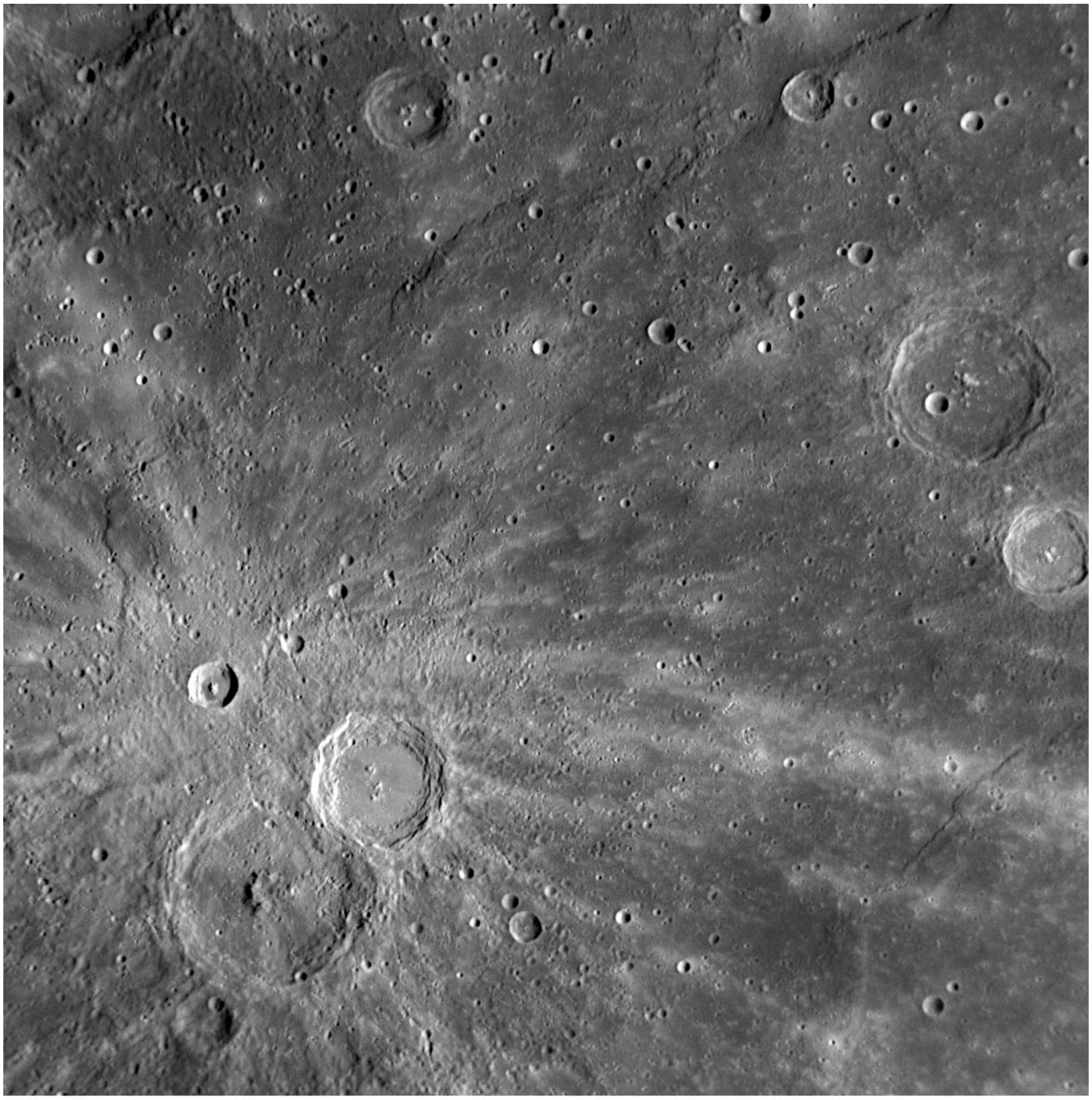
Question 2: How big is the smallest crater in the image, in meters?
Answer: The largest number of features are about 1 millimeter across or 100 meters to one significant figure. Students may also go for the recognizable craters which are about 2 millimeters across or about 200 meters.

Question 3: About what is the typical distance between craters in the image?
Answer: The answer may vary, but the distance between obvious craters (about 2 mm in diameter) is about 5 millimeters or 5 mm x 124 meters/mm = 600 meters to one significant figure.

Question 5: How far would you have to walk between the largest, and next-largest craters?
Answer: The crater rims are about 35 millimeters apart or 35 mm x 124 meters/mm = 4,340 meters or 4.3 kilometers to two significant figures.

Extra for Experts

Image Name - Mercury Craters
Instrument - MESSENGER spacecraft
Image size = 563 kilometers wide
Distance - 19,760 kilometers



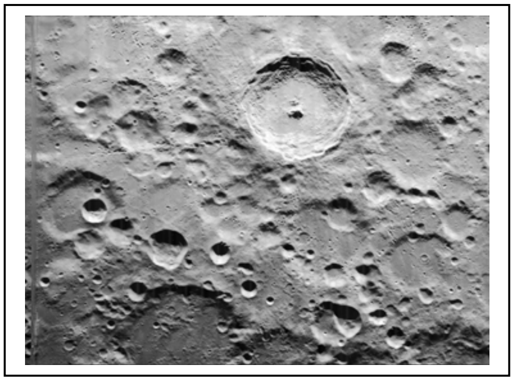
From a GOOGLE search or other resource, what kind of object is this, and what other object does it look similar to?

What is the image scale?

Devise three questions and their answers that explore the contents of the image based on the calculated image scale.

Answer: Mercury Craters - The surface of the planet mercury resembles our very own moon! - Scale = 563 km/153mm = 3.7 km/mm

Lunar Cratering - Probability and Odds



The moon has lots of craters! If you look carefully at them, you will discover that many overlap each other. Suppose that over a period of 100,000 years, four asteroids struck the lunar surface. What would be the probability that they would strike an already-cratered area, or the lunar mare, where there are few craters?

Problem 1 - Suppose you had a coin where one face was labeled 'C' for cratered and the other labeled U for uncratered. What are all of the possibilities for flipping C and U with four coin flips?

Problem 2 - How many ways can you flip the coin and get only Us?

Problem 3 - How many ways can you flip the coin and get only Cs?

Problem 4 - How many ways can you flip the coin and get 2 Cs and 2 Us?

Problem 5 - Out of all the possible outcomes, what fraction includes only one 'U' as a possibility?

Problem 6 - If the fraction of desired outcomes is 2/16, which reduces to 1/8, we say that the 'odds' for that outcome are 1 chance in 8. What are the odds for the outcome in Problem 4?

A fair coin is defined as a coin whose two sides have equal probability of occurring so that the probability for 'heads' = 1/2 and the probability for tails = 1/2 as well. This means that P(heads) + P(tails) = 1/2 + 1/2 = 1. Suppose a tampered coin had P(heads) = 2/3 and P(tails) = 1/3. We would still have P(heads) + P(tails) = 1, but the probability of the outcomes would be different…and in the cheater's favor. For example, in two coin flips, the outcomes would be HH, HT ,TH and TT but the probabilities for each of these would be HH = (2/3)x(2/3)=4/9; HT and TH = 2 x (2/3)(1/3) = 4/9, and TT = (1/3)x1/3) = 1/9. The probability of getting more heads would be 4/9 + 4/9 = 8/9 which is much higher than for a fair coin.

Problem 7: From your answers to Problem 2, what would be the probability of getting only Us in 4 coin tosses if A) P(U) = 1/2? B) P(U) = 1/3?

Problem 8 - The fraction of the lunar surface that is cratered is 3/4, while the mare (dark areas) have few craters and occupy 1/4 of the surface area. If four asteroids were to strike the moon in 100,000 years, A) what is the probability that all four would strike the cratered areas?

Answer Key

Problem 1 - The 16 possibilities are as follows:

C U U U	C C U U	U C U C	C U C C
U C U U	C U C U	U U C C	U C C C
U U C U	C U U C	C C C U	C C C C
U U U C	U C C U	C C U C	U U U U

Note if there are two outcomes for each coin flip, there are 2 x 2 x 2 x 2 = 16 independent possibilities.

Problem 2 - There is only one outcome that has 'U U U U'

Problem 3 - There is only one outcome that has 'C C C C'

Problem 4 - From the tabulation, there are 6 ways to get this outcome in any order.

Problem 5 - There are 4 outcomes that have only one U out of the 16 possibile outcomes, so the fraction is 4/16 or 1/4.

Problem 6 - The fraction is 6 / 16 reduces to 3/4 so the odds are 3 chances in 4.

Problem 7: A) If each U has a probability of 1/2, then the probability is 1/16 x (1/2) x (1/2) x (1/2) x (1/2) = 1/(16 x 16) =1/256.

B) If each U has a probability of 1/3, then the probability is 1/16 x (1/3) x (1/3) x (1/3) x (1/3) = 1/(16 x 81) = 1/1296.

Problem 8 - P(U) = 1/4 while P(C)= 3/4, so the probability that all of the impacts are in the uncratered regions is the outcome C C C C which is 1/16 of all possible outcomes, so its probability is 1/16 x (3/4) x (3/4) x (3/4) x (3/4) = 81 / 4096 = 0.0198.

Lunar Meteorite Impact Risks

A December 4, 2006 CNN.Com news story, based on the research by Bill Cooke, head of NASA's Meteoroid Environment Office suggests that one of the largest dangers to lunar explorers will be meteorite impacts. Between November 2005 and November 2006, Dr. Cooke's observations of lunar flashes (see image) found 12 of these events in a single year. The flashes were caused primarily by Leonid Meteors about 3-inches across, impacting with the equivalent energy of 150-300 pounds of TNT.

The diameter of the moon is 3,476 kilometers.

Problem 1: From the formula for the surface of a sphere, what is the area, in square kilometers, of the side of the moon facing Earth?

Problem 2: Although an actual impact only affects the few square meters within its immediate vicinity, we can define an impact zone area as the total area of the surface being struck, by the number of objects striking it. What was the average impact zone area for a single event?

Problem 3: Assuming the area is a square with a side length 'S', A) what is the length of the side of the impact area? B) What is the average distance between the centers of each impact area?

Problem 4: If the impacts happen randomly and uniformly in time, about what would be the time interval between impacts?

Problem 5: From the vantage point of an astronaut standing on the Moon, the horizon is about 3 kilometers away. How long would the lunar colony have to wait before it was likely to see an impact within its horizon area?

Problem 6: The lunar image shows that the impacts are not really random, but seem clustered into three groups. Each group covers an area about 700 kilometers on a side. What is the average impact zone area for four strikes per zone?

Problem 7: If you were an colony located in one of these three zones, what would be your answer to Problem 5?

Answer Key:

Problem 1: From the formula for the surface of a sphere, A) what is the area, in square kilometers, of the side of the moon facing Earth?

Answer: $2 \times 3.141 \times (1738\ km)^2 = 1.89 \times 10^7\ km^2$

Problem 2: Answer: $1.89 \times 10^7\ km^2 /12 = 1.58 \times 10^6\ km^2$

Problem 3: Assuming the area is a square with a side length 'S', A) what is the length of the side of the impact area? B) What is the average distance between the centers of each impact area?

Answer: A) $S = (1.58 \times 10^6\ km^2)^{1/2}$ about 1,257 kilometers B) 1,257 kilometers.

Problem 4: If the impacts happen randomly and uniformly in time, about what would be the time interval between impacts?

Answer: 1 year / 12 impacts = One month.

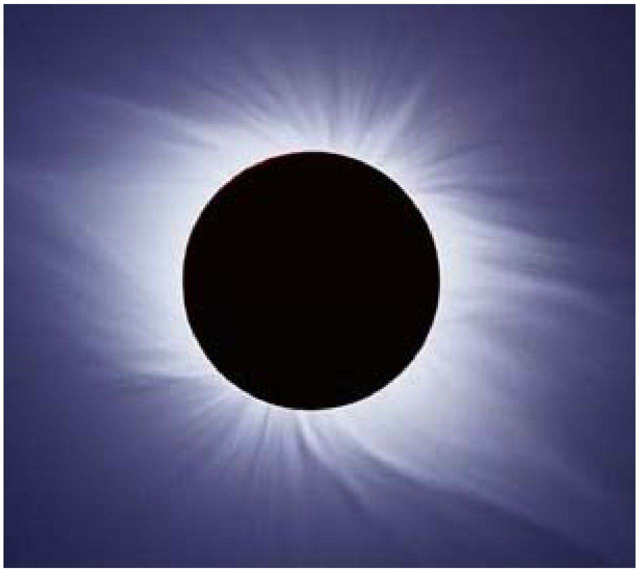
Problem 5: From the vantage point of an astronaut standing on the Moon, the horizon is about 3 kilometers away. How long would the lunar colony have to wait before it was likely to see an impact within its horizon area?

Answer: 1 impact per 1.58×10^6 square kilometers per month. The area of the horizon region around the colony is about $\pi \times (3km)^2 =$ 27 square kilometers. This area is $1.58 \times 10^6\ km^2 /27\ km^2$ or about 60,000 times smaller that the average, monthly impact area. That suggests you will have to wait about 60,000 times longer than the time it takes for one impact or 60,000 months, which equals 5,000 years, assuming that the distribution of impacts is completely random, unbiased and has a uniform geographic distribution across the Moon's surface.

Problem 6: Answer: (700 km) x (700 km) / 4 = 1 impact per 122,500 km^2 zone area. Horizon area = 27 km^2, so the impact zone area is 122,500 / 27 = 4,500 times larger. You would need to wait about 4,500 x 1 month or 375 years for an impact to happen within your horizon.

Note to Teacher: This calculation assumes that the clustering of impacts is a real effect that persists over a long time. In fact, this is very unlikely, and it is more statistically probable that when thousands of impacts are plotted, a more uniform strike distribution will result. This is similar to the result of flipping a coin 12 times and getting a different outcome than half-Heads and half-Tails.

The Last Total Solar Eclipse...Ever!



Total solar eclipses happen because the angular size of the moon is almost exactly the same as the sun's, despite their vastly different distances and sizes.

The moon has been steadily pulling away from earth over the span of billions of years. There will eventually come a time when these two angular sizes no longer match up. The moon will be too small to cause a total solar eclipse.

When will that happen?

Image courtesy Fred Espenak
http://sunearth.gsfc.nasa.gov/eclipse/eclipse.html

Problem 1 - The minimum distance to the moon, called the perigee, is 356,400 kilometers. At that distance, the angular size of the moon from the surface of Earth is 0.559 degrees. Suppose you doubled the distance to the moon. What would its new angular size be, as seen from the surface of Earth?

Problem 2 - Suppose you increased the moon's distance by 50,000 kilometers. What would the angular size now be?

Problem 3 - The smallest angular size of the sun occurs near the summer solstice at a distance of 152 million kilometers, when the sun has an angular diameter of 0.525 degrees. How far away, in kilometers, does the moon have to be to match the sun's apparent diameter?

Problem 4 - How much further away from Earth will the moon be at that time?

Problem 5: The moon is moving away from Earth at a rate of 3 centimeters per year. How many years will it take to move 3 kilometer further away?

Problem 6: How many years will it take to move the distance from your answer to Problem 4?

Problem 7: When will the last Total Solar Eclipse be sighted in the future?

Answer Key:

Problem 1 - The minimum distance to the moon, called the perigee, is 356,400 kilometers. At that distance, the angular size of the moon from the surface of Earth is 0.559 degrees. Suppose you doubled the distance to the moon. What would its new angular size be, as seen from the surface of Earth?

Answer: Because objects appear smaller the farther away they are, if you double the distance, the moon will appear half its former size, or 0.559/2 = 0.279 degrees across.

Problem 2 - Suppose you increased the moon's distance by 50,000 kilometers. What would the angular size now be?

Answer: The distance is now 356,400 kilometers + 50,000 kilometers = 406,400 kilometers. The distance has increased by 406,400/356,400 = 1.14, so that means that the angular size has been reduced to 0.559 / 1.14 = 0.49 degrees.

Problem 3 - The smallest angular size of the sun occurs near the summer solstice at a distance of 152 million kilometers, when the sun has an angular diameter of 0.525 degrees. How far away, in kilometers, does the moon have to be to match the sun's apparent diameter?

Answer: 0.559/0.525 = 1.06 times further away from Earth or 356,400 km x 1.06 = 377,800 kilometers.

Problem 4 - How much further away from Earth will the moon be at that time?

Answer: 377,800 kilometers - 356,400 kilometers = 21,400 kilometers.

Problem 5: The moon is moving away from Earth at a rate of 3 centimeters per year. How many years will it take to move 3 kilometer further away?

Answer: (300,000 centimeters) / (3 centimeters / year) = 100,000 years.

Problem 6: How many years will it take to move the distance from your answer to Problem 4?

Answer: (21,400 kilometers / 3 kilometers) x 100,000 years = 713 million years.

Problem 7: When will the last Total Solar Eclipse be sighted in the future?

Answer: About 713 million years from now.

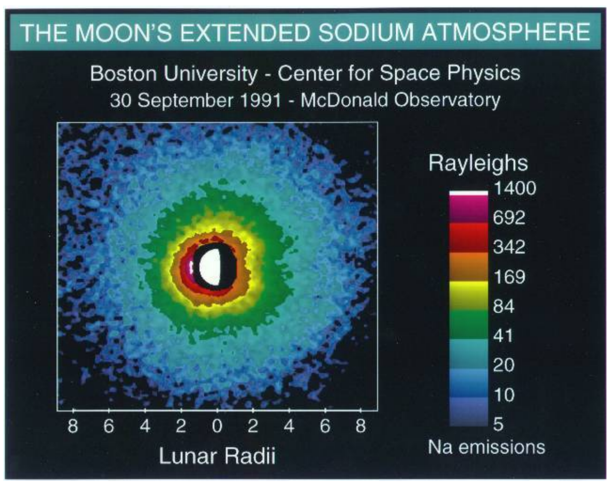
The Moon's Atmosphere!

Courtesy: T.A.Rector, I.P.Dell'Antonio (NOAO/AURA/NSF)

Experiments performed by Apollo astronauts were able to confirm that the moon does have a very thin atmosphere.

The Moon has an atmosphere, but it is very tenuous. Gases in the lunar atmosphere are easily lost to space. Because of the Moon's low gravity, light atoms such as helium receive sufficient energy from solar heating that they escape in just a few hours. Heavier atoms take longer to escape, but are ultimately ionized by the Sun's ultraviolet radiation, after which they are carried away from the Moon by solar wind.

Because of the rate at which atoms escape from the lunar atmosphere, there must be a continuous source of particles to maintain even a tenuous atmosphere. Sources for the lunar atmosphere include the capture of particles from solar wind and the material released from the impact of comets and meteorites. For some atoms, particularly helium and argon, outgassing from the Moon's interior may also be a source.



Problem 1: The Cold Cathode Ion Gauge instrument used by Apollo 12, 14 and 15 recorded a daytime atmosphere density of 160,000 atoms/cc of hydrogen, helium, neon and argon in equal proportions. What was the density of helium in particles/cc?

Problem 2: The atomic masses of hydrogen, helium, neon and argon are 1.0 AMU, 4.0 AMU, 20 AMU and 36 AMU. If one AMU = 1.6×10^{-24} grams, a) How many grams of hydrogen are in one cm3 of the moon's atmosphere? B) Helium? C) Neon? D) Argon? E) Total grams from all atoms?

Problem 3: Assume that the atmosphere fills a spherical shell with a radius of 1,738 kilometers, and a thickness of 170 kilometers. What is the volume of this spherical shell in cubic centimeters?

Problem 4. Your answer to Problem 2E is the total density of the lunar atmosphere in grams/cc. If the atmosphere occupies the shell whose volume is given in Problem 3, what is the total mass of the atmosphere in A) grams? B) kilograms? C) metric tons?

Answer Key:

Problem 1: The Cold Cathode Ion Gauge instrument used by Apollo 12, 14 and 15 recorded a daytime atmosphere density of 160,000 atoms/cc of hydrogen, helium, neon and argon in equal proportions. What was the density of helium in particles/cc?

Answer: Each element contributes 1/4 of the total particles so hydrogen = 40,000 particles/cc; helium = 40,000 particles/cc, argon=40,000 particles/cc and argon=40,000 particles/cc

Problem 2: The atomic masses of hydrogen, helium, neon and argon are 1.0 AMU, 4.0 AMU, 20 AMU and 36 AMU. If one AMU = 1.6 x 10-24 grams, a) How many grams of hydrogen are in one cm3 of the moon's atmosphere? B) Helium? C) Neon? D) Argon? E) Total grams from all atoms?

Answer: A) Hydrogen = 1.0 x (1.6 x 10^{-24} grams) x 40,000 particles = 6.4 x $10^{\mathbf{-20}}$ grams
B) Helium = 4.0 x (1.6 x 10^{-24} grams) x 40,000 particles = 2.6 x $10^{\mathbf{-19}}$ grams
C) Neon = 20.0 x (1.6 x 10^{-24} grams) x 40,000 particles = 1.3 x $10^{\mathbf{-18}}$ grams
D) Argon = 36.0 x (1.6 x 10^{-24} grams) x 40,000 particles = 2.3 x $10^{\mathbf{-18}}$ grams
E) Total = (0.064 + 0.26 + 1.3 + 2.3) x 10^{-18} grams = <u>3.9 x $10^{\mathbf{-18}}$ grams per cc.</u>

Problem 3: Assume that the atmosphere fills a spherical shell with a radius of 1,738 kilometers, and a thickness of 170 kilometers. What is the volume of this spherical shell in cubic centimeters?

Answer: Compute the difference in volume between A sphere with a radius of Ri = 1,738 km and Ro = 1,738+170 = 1,908 km. V = 4/3 π $(1908)^{\mathbf{3}}$ - 4/3π $(1738)^{\mathbf{3}}$ = 2.909 x $10^{\mathbf{10}}$ $km^{\mathbf{3}}$ - 2.198 x $10^{\mathbf{10}}$ $km^{\mathbf{3}}$ = 7.1 x $10^{\mathbf{9}}$ $km^{\mathbf{3}}$

Volume = 7.1 x $10^{\mathbf{9}}$ $km^{\mathbf{3}}$ **x** (10^5 cm/km) **x** (10^5 cm/km) **x** (10^5 cm/km)
= 7.1 x 10^{24} cm^3

Note: If you use the 'calculus technique' of approximating the volume as the surface area of the shell with a radius of Ri, multiplied by the shell thickness of h = 170 km, you will get a slightly different answer of 6.5 x $10^{\mathbf{9}}$ $km^{\mathbf{3}}$ and 6.5 x 10^{24} cm^3

Problem 4. Your answer to Problem 2E is the total density of the lunar atmosphere in grams/cc. If the atmosphere occupies the shell whose volume is given in Problem 3, what is the total mass of the atmosphere in A) grams? B) kilograms?

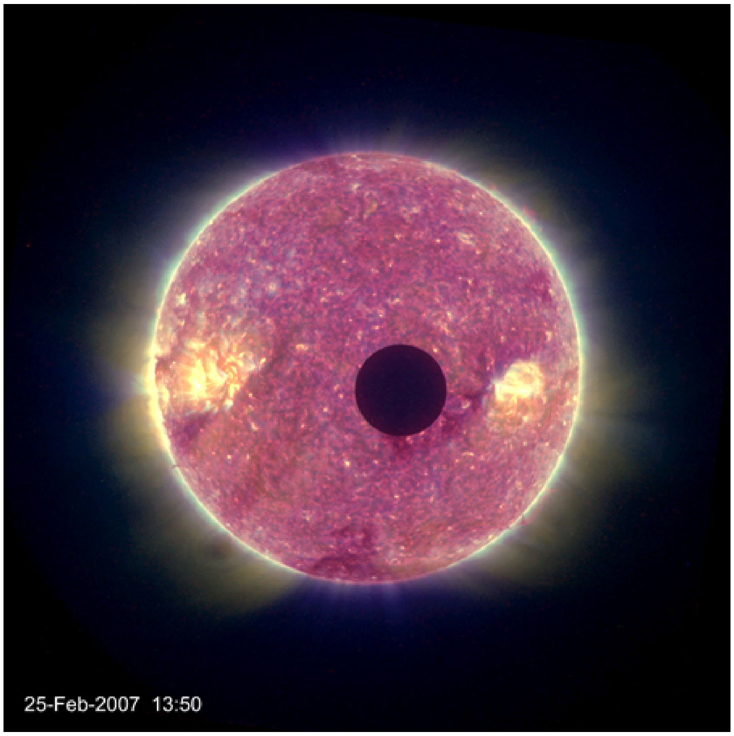
A) Mass = density x volume = (3.9 x $10^{\mathbf{-18}}$ gm/cc) x 7.1 x $10^{\mathbf{24}}$ $cm^{\mathbf{3}}$ = 2.8 x $10^{\mathbf{7}}$ grams

B) Mass = 2.8 x $10^{\mathbf{7}}$ grams x (1 kg/1000 gms) = 28,000 kilograms.

C) Mass = 28,000 kg x (1 ton / 1000 kg) = 28 tons.

Teacher note: You may want to compare this mass to some other familiar objects. Also, the Apollo 11 landing and take-off rockets ejected about 1 ton of exhaust gases. Have the students discuss the human impact (air pollution!) on the lunar atmosphere from landings and launches.

A Lunar Transit of the Sun from Space



The twin STEREO satellites captured this picture of our Moon passing across the sun's disk on February 25, 2007. The two satellites are located approximately in the orbit of Earth, but are moving away from Earth in opposite directions. From this image, you can figure out how far away from the Moon the STEREO-B satellite was when it took this picture! To do this, all you need to know is the following:

1) The diameter of the Moon is 3,476 km

2) The distance to the Sun is 150 million km.

3) The diameter of the Sun is 0.54 degrees

Can you figure out how to do this using geometry?

Problem 1: Although the True Size of an object is measured in meters or kilometers, the Apparent Size of an object is measured in terms of the number of angular degrees it subtends. Although the True Size of an object remains the same no mater how far away it is from you, the Apparent Size gets smaller the further away it is. In the image above, the Apparent Size of the Sun was 0.54 degrees across on February 25. By using a millimeter ruler and a calculator, what is the angular size of the Moon?

Problem 2: As seen from the distance of Earth, the Moon has an Apparent Size of 0.53 degrees. If the Earth-Moon distance is 384,000 kilometers, how big would the Moon appear at twice this distance?

Problem 3: From your answer to Problem 1, and Problem 2, what is the distance to the Moon from where the above photo was taken by the STEREO-B satellite?

Problem 4: On February 25, 2007 there was a Half Moon as viewed from Earth, can you draw a scaled model of the Earth, Moon, Stereo-B and Sun distances and positions (but not diameters to the same scale!) with a compass, ruler and protractor?

Answer Key:

Problem 1: Although the True Size of an object is measured in meters or kilometers, the Apparent Size of an object is measured in terms of the number of angular degrees it subtends. Although the True Size of an object remains the same no mater how far away it is from you, the Apparent Size gets smaller the further away it is. In the image above, the Apparent Size of the sun is 0.5 degrees across. By using a millimeter ruler and a calculator, what is the angular size of the Moon?

Answer: The diameter of the sun is 57 millimeters. This represents 0.54 degrees, so the image scale is 0.54 degrees / 57 millimeters = 0.0095 degrees/mm
The diameter of the Moon is 12 millimeters, so the angular size of the Moon is

12 mm x 0.0095 degrees/mm = 0.11 degrees.

Problem 2: As seen from the distance of Earth, the Moon has an Apparent Size of 0.53 degrees. If the Earth-Moon distance is 384,000 kilometers, how big would the Moon appear at twice this distance?

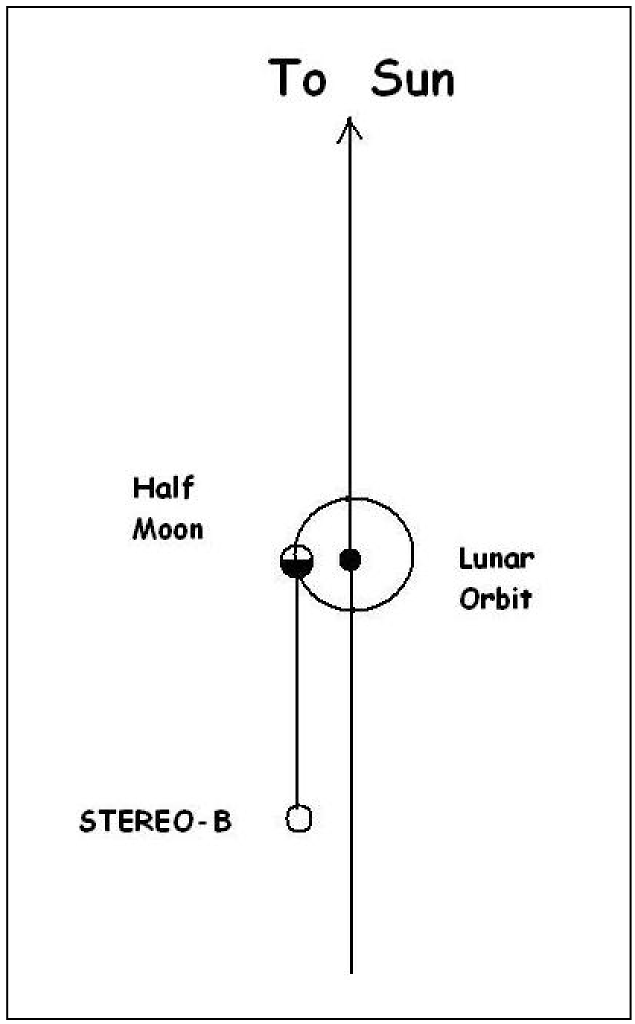
Answer: It would have an Apparent Size half as large, or 0.26 degrees.

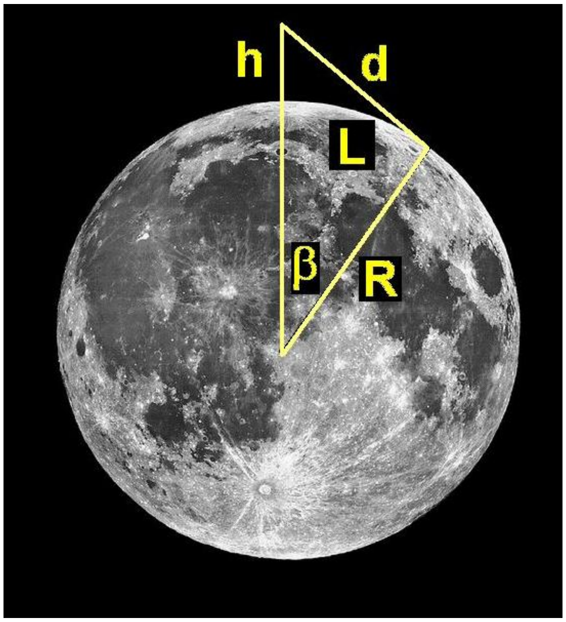
Problem 3: From your answer to Problem 1, and Problem 2, what is the distance to the moon from where the above photo was taken by the STEREO-B satellite?

Answer: The ratio of the solar diameter to the lunar diameter is 0.54 degrees/0.11 degrees = 4.9. This means that from the vantage point of STEREO, it is 4.9 times farther away than it would be at the Earth-Moon distance. This means it is 4.9 times farther away than 384,000 km, or 1.9 million kilometers.

Problem 4: On February 25, 2007 there was a Half Moon as viewed from Earth, can you draw a scaled model of the Earth, Moon, Stereo-B and Sun distances and positions (but not diameters!) using a compass, ruler and protractor?

Answer: The figure to the right shows the locations of the Earth, Moon and STEREO satellite. The line connecting the Moon and the Satellite is 4.9 times the Earth-Moon distance.





An important quantity in planetary exploration is the distance to the horizon. This will, naturally, depend on the diameter of the planet (or asteroid!) and the height of the observer above the ground.

Another application of this geometry is in determining the height of a transmission antenna in order to insure proper reception out to a specified distance. This is especially important for lunar explorers because the Moon does not have an ionosphere capable of 'bouncing' the radio signals over the horizon.

Problem 1: If the radius of the planet is given by R, and the height above the surface is given by h, use the figure above, and the Pythagorean Theorem, to derive the formula for the line-of-sight horizon distance, d, to the horizon tangent point.

Problem 2: Derive the distance along the planet, L, to the tangent point.

Problem 3: For a typical human height of 2 meters, what is the horizon distance on A) Earth (R=6378 km); B) The Moon (1,738 km)

Problem 4: A radio station has an antenna tower 50 meters tall. What is the maximum line-of-sight (LOS) reception distance on the Moon?

Answer Key:

Problem 1: If the radius of the planet is given by R, and the height above the surface is given by h, use the figure to the left to derive the formula for the line-of-sight horizon distance, D.

Answer: By the Pythagorean Theorem $d^2 = (R+h)^2 - R^2$

so $d = (R^2 + 2Rh + h^2 - R^2)^{1/2}$ and so the answer is $d = (h^2 + 2Rh)^{1/2}$

Problem 2: Derive the distance along the planet, l, to the tangent point.

Answer: From the diagram, $\cos(\beta) = R/(R+h)$ and so $L = R \arccos(R/(R+h))$

Problem 3: For a typical human height of 2 meters, what is the horizon distance on A) Earth (R=6,378 km); B) The Moon (1,738 km)

Answer: Use the equation from Problem 1.

A) R=6378 km and h=2 meters so

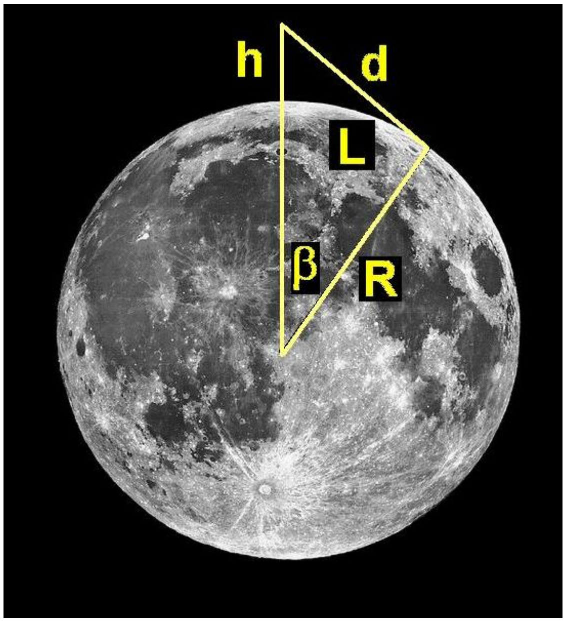
$d = ((2 \text{ meters})^2 + 2 \times 2 \text{ meters} \times 6.378 \times 10^6 \text{ meters})^{1/2}$ = 5051 meters or 5.1 kilometers.

B) For the Moon, R=1,738 km so d = 2.6 kilometers

Problem 4: A radio station has an antenna tower 50 meters tall. What is the maximum line-of-sight (LOS) reception distance on the Moon?

Answer: A) h = 50 meters, R=1,738 km so d = 13,183 meters or 13.2 kilometers.

Beyond the Blue Horizon.... Advanced



An important quantity in planetary exploration is the distance to the horizon. This will, naturally, depend on the diameter of the planet (or asteroid!) and the height of the observer above the ground.

Another application of this geometry is in determining the height of a transmission antenna on the Moon in order to insure proper reception out to a specified distance.

Teachers: *Problems 1-4 can be successfully accomplished by algebra students. Problems 5 and 6 require a knowledge of derivatives and can be assigned to calculus students after they have completed Problem 1 and 2.*

Problem 1: If the radius of the planet is given by R, and the height above the surface is given by h, use the figure above to derive the formula for the line-of-sight (LOS) horizon distance, d, to the horizon tangent point.

Problem 2: Derive the distance along the planet, L, to the tangent point.

Problem 3: For a typical human height of 2 meters, what is the horizon distance on A) Earth (R=6,378 km); B) The Moon (1,738 km)

Problem 4: A radio station has an antenna tower 50 meters tall. What is the maximum line-of-sight (LOS) reception distance in the Moon?

Problem 5: What is the rate of change of the lunar LOS radius, d, for each additional meter of antenna height in Problem 4?

Problem 6: What is the rate-of-change of the distance to the lunar radio tower, L, at the LOS position in Problem 4?

Answer Key:

Problem 1: By the Pythagorean Theorem $d^2 = (R+h)^2 - R^2$

so $d = (R^2 + 2Rh + h^2 - R^2)^{1/2}$

and so $d = (h^2 + 2Rh)^{1/2}$

Problem 2: Derive the distance along the planet, l, to the tangent point. From the diagram,

$\cos(\beta) = R/(R+h)$ and so $L = R \operatorname{Arccos}(R/(R+h))$

Problem 3: Use the equation from Problem 1. A) For Earth, R=6378 km and h=2 meters so $d = ((2 \text{ meters})^2 + 2 \times 2 \text{ meters} \times 6.378 \times 10^6 \text{ meters}))^{1/2}$ = 5051 meters or 5.1 kilometers.
B) For the Moon, R=1,738 km so d = 2.6 kilometers

Problem 4: h = 50 meters, R=1,738 km so d = 13,183 meters or 13.2 kilometers.

Problem 5: Use the chain rule to take the derivative with respect to h of the equation for d in Problem 1. Evaluate dd/dh at h=50 meters for R=1,738 km.

Let $U = h^2 + 2Rh$ then $d = U^{1/2}$ so $dU/dh = (dd/dU)(dU/dh)$

Then $dd/dh = +1/2\ U^{-1/2}$

$dU/dh = +1/2\ (2h + 2R)(h^2 + 2Rh)^{-1/2}$

For h=50 meters and R = 1,738 km,

$dd/dh = +0.5 \times (100 + 3476000)(2500 + 2 \times 50 \times 1738000)^{-1/2}$

= +131.8 meters in LOS distance per meter of height.

Problem 6: Let $U = R/(R+h)$, then $L = R \cos^{-1}(U)$.

By the chain rule $dL/dh = (dL/dU) \times (dU/dh)$.

Since $dL/dU = R \times (-1)(1 - u2)^{1/2}$ and $dU/dh = R \times (-1) \times (R + h)^{-2}$ then

$dL/dh = R^2\ (R + h)^{-2}\ (R + h)^{1/2}\ /((R+ h)^2 - R^2)^{1/2}$

$dL/dh = R^2 (R+h)^{-1} (h^2 + 2Rh)^{-1/2}$

Since R >> h, $dL/dh = R/(2Rh)^{1/2}$

Evaluating this for R = 1,738 km and h = 50 meters gives dL/dh = +131.8 meters per kilometer.

The Mass of the Moon

On July 19, 1969 the Apollo-11 Command Service Module and LEM entered lunar orbit. The orbit period was 2.0 hours, at a distance of 1,737 kilometers from the lunar center.

Believe it or not, you can use these two pieces of information to determine the mass of the moon. Here's how it's done!

Problem 1 - Assume that Apollo-11 went into a circular orbit, and that the inward gravitational acceleration by the moon on the capsule, Fg, exactly balances the outward centrifugal acceleration, Fc. Solve Fc = Fg for the mass of the moon, M, in terms of V, R and the constant of gravity, G, given that:

$$F_g = \frac{G\,M\,m}{R^2} \qquad F_c = \frac{m\,V^2}{R}$$

Problem 2 - By using the fact that for circular motion, $V = 2\pi R / T$, re-express your answer to Problem 1 in terms of R, T and M.

Problem 3 - Given that $G = 6.67 \times 10^{-11}\ m^3\ kg^{-1}\ sec^{-2}$, R = 1,737 kilometers and T = 2 hours, calculate the mass of the moon in kilograms!

Problem 4 - The mass of Earth is 5.97×10^{24} kilograms. What is the ratio of the moon's mass, derived in Problem 3, to Earth's mass?

Answer Key

Problem 1 - From Fg = Fc, and a little algebra to simplify and cancel terms, you get

$$M = \frac{R V^2}{G}$$

Problem 2 - Substitute 2 π R / T for V and with little algebra to simplify and cancel terms, you get :

$$M = \frac{4\pi^2 R^3}{G T^2}$$

Problem 3 - First convert all units to meters and seconds: R = 1.737×10^6 meters and T = 7,200 seconds. Then substitute values into the above equation:

$M = 4 \times (3.14)^2 \times (1.737 \times 10^6)^3 / (6.67 \times 10^{-11} \times (7200)^2))$

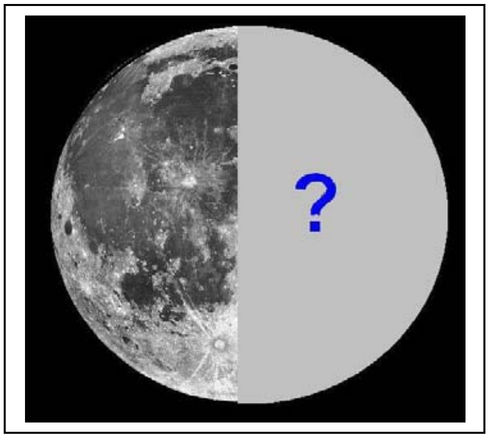
$M = (39.44 \times 5.24 \times 10^{18}) / (3.46 \times 10^{-3})$

M = 5.97×10^{22} kilograms

More accurate measurements, allowing for the influence of Earth's gravity and careful timing of orbital periods, actually yield 7.4×10^{22} kilograms.

Problem 4 - The ratio of the masses is 5.97×10^{22} kilograms / 5.97×10^{24} kilograms which equals 1/100. The actual mass ratio is 1 / 80.

The Moon's Density - What's inside?



The Moon has a mass of 7.4×10^{22} kilograms and a radius of 1,737 kilometers. Seismic data from the Apollo seismometers also shows that there is a boundary inside the Moon at a radius of about 400 kilometers where the rock density or composition changes. Astronomers can use this information to create a model of the Moon's interior.

Problem 1 - What is the average density of the Moon in grams per cubic centimeter (g/cm^3) ? (Assume the Moon is a perfect sphere.)

Problem 2 - What is the volume, in cubic centimeters, of A) the Moon's interior out to a radius of 400 km? and B) The remaining volume out to the surface?

You can make a simple model of a planet's interior by thinking of it as an inner sphere (the core) with a radius of R(core), surrounded by a spherical shell (the mantle) that extends from R(core) to the planet's surface, R(surface). We know the total mass of the planet, and its radius, R(surface). The challenge is to come up with densities for the core and mantle and R(core) that give the total mass that is observed.

Problem 3 - From this information, what is the total mass of the planet model in terms of the densities of the two rock types (D1 and D2) and the radius of the core and mantle regions R(core) and R(surface)?

Problem 4 - The densities of various rock types are given in the table below.

Type	Density
I - Iron+Nickle mixture (Earth's core)	15.0 gm/cc
E - Earth's mantle rock (compressed)	4.5 gm/cc
B - Basalts	2.9 gm/cc
G - Granite	2.7 gm/cc
S - Sandstone	2.5 gm/cc

A) How many possible lunar models are there? B) List them using the code letters in the above table, C) If denser rocks are typically found deep inside a planet, which possibilities survive? D) Find combinations of the above rock types for the core and mantle regions of the lunar interior model, that give approximately the correct lunar mass of 7.4×10^{25} grams. [Hint: use an *Excel* spread sheet to make the calculations faster as you change the parameters.] E) If Apollo rock samples give an average surface density of 3.0 gm/cc, which models give the best estimates for the Moon's interior structure?

Answer Key

Problem 1 - Mass = 7.4×10^{22} kg x 1000 gm/kg = 7.4×10^{25} grams. Radius = 1,737 km x 100,000 cm/km = 1.737×10^{8} cm. Volume of a sphere = $4/3\ \pi\ R^3$ = 4/3 x (3.141) x $(1.737 \times 10^{8})^3$ = $2.2 \times 10^{25}\ cm^3$, so the density = 7.4×10^{25} grams / $2.2 \times 10^{25}\ cm^3$ = $3.4\ gm / cm^3$.

Problem 2 - A) V(core) = $4/3\ \pi\ R^3$ = 4/3 x (3.141) x $(4.0 \times 10^{7})^3$ = $2.7 \times 10^{23}\ cm^3$
B) V(shell) = V(Rsurface) - V(Rcore) = $2.2 \times 10^{25}\ cm^3$ - $2.7 \times 10^{23}\ cm^3$ = $2.17 \times 10^{25}\ cm^3$

Problem 3 - The total core mass is given by M(core) = $4/3\ \pi\ (Rcore)^3$ x D1. The volume of the mantle shell is given by multiplying the shell volume V(shell) calculated in Problem 2B by the density : Mshell = V(shell) x D2. Then, the formula for the total mass of the model is given by: MT = $4/3\ \pi\ (Rc)^3$ x D1 + ($4/3\ \pi\ (Rs)^3 - 4/3\ \pi\ (Rc)^3$) x D2, which can be simplified to:

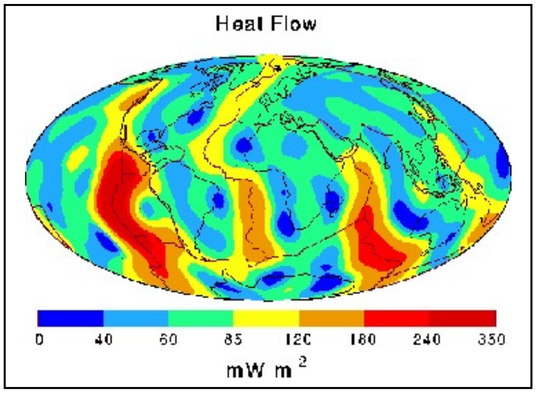
$$MT = 4/3\ \pi\ (D1 \times Rc^3 + D2 \times Rs^3 - D2 \times Rc^3)$$

Problem 4 - A) There are 5 types of rock for 2 lunar regions so the number of unique models is 5 x 5 = 25 possible models. B) The possibilities are: II, IE, IB, IG, IS, EE, EI, EB, EG, ES, BI, BE, BB, BG, BS, GI ,GE, GB, GG, GS, SI, SE, SB, SG, SS. C) The ones that are physically reasonable are: IE, IB, IG, IS, EB, EG, ES, BG, BS, GS. The models, II, EE, BB, GG and SS are eliminated because the core must be denser than the mantle. D) Each possibility in your answer to Part C has to be evaluated by using the equation you derived in Problem 3. This can be done very efficiently by using an Excel spreadsheet. The possible answers are as follows:

Model Code	Mass (in units of 10^{25} grams)
I E	10.2
I B	6.7
E B	6.4
I G	6.3
E G	6.0
B G	6.0
I S	5.8
E S	5.5
B S	5.5
G S	5.5

E) The models that have rocks with a density near 3.0 gm/cc as the mantle top layer are the more consistent with the density of surface rocks, so these would be IB and EB which have mass estimates of 6.7×10^{25} and 6.4×10^{25} grams respectively. These are both very close to the actual moon mass of 7.4×10^{25} grams (e.g. 7.4×10^{22} kilograms) so it is likely that the moon has an outer mantle consisting of basaltic rock, similar to Earth's mantle rock (4.5 gm/cc) and a core consisting of a denser iron/nickel mixture (15 gm/cc).

The Hot Lunar Interior



Earth heat flow map (H. N. Pollack, S. J. Hurter, and J. R. Johnson, Reviews of Geophysics, Vol. 31, 1993.)

When large bodies form, their interiors are heated by a combination of radioactivity and the heat of formation from the infall of the rock. For planet-sized bodies, this heat can be generated and lost over billions of years. The end result will be that the core cools off and, if molten, it eventually solidifies. Measuring the surface temperature of a solid body, and the rate at which heat escapes its surface, provides clues to its internal heating and cooling rates.

Problem 1 - Measuring the heat flow out of the lunar surface is a challenge because the monthly and annual changes of surface solar heating produce interference. Apollo 15 astronauts measured the heat flow from two bore holes that reached about 2-meters below the surface. When corrected for the monthly effects from the Sun, they detected a heat flow of about 20 milliWatts/meter^2. If the radius of the Moon is 1,737 kilometers, what is the total thermal power emitted by the entire Moon in billions of watts?

Problem 2 - A future lunar colony covers a square surface that is 100 meters x 100 meters. What is the total thermal power available to this colony by 'harvesting' the lunar heat flow?

Problem 3 - The relationship between power, L, surface radius, R, and surface temperature, T, is given by $L = 4 \pi R^2 \sigma T^4$ where σ = the Stefan-Boltzman constant and has a value of 5.67×10^{-8} W/m^2 / K^4 , and where T is in Kelvin degrees, L is in watts, and R is in meters. Suppose the Moon's interior was heated by a source with a radius of 400 kilometers at the lunar core, what would the temperature of this core region have to be to generate the observed thermal wattage at the surface?

Problem 4 - The lunar regolith and crust is a very good insulator! Through various studies, the temperature of the Moon is actually believed to be near 1,200 K within 400 km of the center. A) Using the formula for L in Problem 3, how much power is absorbed by the lunar rock overlaying the core? B) From you answer to (A), how many joules are absorbed by each cubic centimeter of overlaying lunar rock each second (joules/cm^3)? C) Basalt begins to soften when in absorbs over 1 million Joules/cm^3. Is the lunar surface in danger of melting from the heat flow within?

Problem 1 - The surface area of a sphere is $4 \pi R^2$, so the surface area of the moon is $4 \times 3.14 \times (1.737 \times 10^6)^2 = 3.8 \times 10^{18}$ m^2 .The total power, in watts, is then 0.020 watts/m^2 x 3.8×10^{18} m^2 = 7.6×10^{11} watts or 760 billion watts.

Problem 2 - The surface area is 100 x 100 = 10,000 m^2, and with a heat flow of 0.020 milliwatts/m^2, the total thermal power is 200 watts.

Problem 3 - The total thermal power, $L = 7.6 \times 10^{11}$ watts, and R = 400 km = 4.0 x 105 meters, so that $7.6 \times 10^{11} = 4 \times (3.14) \times (4.0 \times 10^5)^2 \times 5.67{\times}10^{-8}\ T^4$
Then $7.6 \times 10^{11} = 1.1 \times 10^5\ T^4$. Solving for T we get T = 51 K.

Problem 4 - A) The power emitted by the 400 km, 1,200 K core region is given by

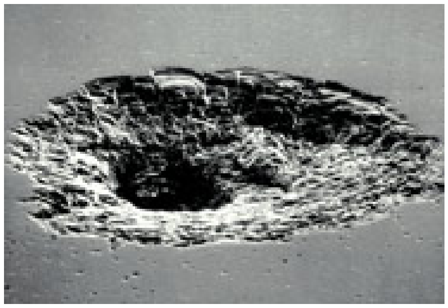
$$L = 4 \pi R^2 \sigma T^4$$

which equals $L = 4 \times (3.14) \times (4.0 \times 10^5)^2 \times 5.67{\times}10^{-8}\ (1200)^4 = 2.4 \times 10^{17}$ watts. Since from the answer to Problem 1 the amount that makes it to the surface is only 7.6×10^{11} watts, that leaves essentially all of the 2.4×10^{17} watts to be absorbed by the overlaying rock mantle.

B) The volume of overlaying rock is the total volume of the moon (radius 1,737 km) minus the volume of the 400-km lunar core. The difference in these two spherical regions is: $4/3\ \pi\ ((1.737 \times 10^6)^3 - (4.0 \times 10^5)^3) = 4/3 \times 3.14 \times (5.18 \times 10^{18}) = 2.2 \times 10^{19}$ m^3, or 2.2×10^{25} cm^3. Since 1 watt = 1 Joule/sec, then in 1 second the lunar thermal power from the 1,200 K core is 2.4×10^{17} joules as calculated in (A). If this is evenly absorbed by the rock in the mantle, the average thermal heating energy per cm^3 is just 2.4×10^{17} joules / 2.2×10^{25} cm^3. or 1.0×10^{-6} joules/cm^3. (This is also equal to 10 ergs /cm^3)

C) No, because this amount of energy input is completely negligible in melting, or warming, rock material. Note: Basalt softens at 1,200 C = 1,500 K. A cubic centimeter of this rock has a surface area of 6 cm^2, so from $L = SA \times \sigma T^4$ we get $L = 6.0 \times 5.67{\times}10^{-8}\ (1500)^4 = 1.7 \times 10^6$ watts, which in 1 second amounts to 1.7×10^6 Joules / cm^3 - the energy needed to melt basalt.

Is there a lunar meteorite hazard?



Damage to Space Shuttle Endeavor in 2000 from a micrometeoroid or debris impact . The crater is about 1mm across. (Courtesy - JPL/NASA)

Without an atmosphere, there is nothing to prevent millions of pounds a year of rock and ice fragments from raining down upon the lunar surface.

Traveling at 10,000 miles per hour (19 km/s), they are faster than a speeding bullet and are utterly silent and invisible until they strike.

Is this something that lunar explorers need to worry about?

Problem 1 - Between 1972 and 1992, military infra-sound sensors on Earth detected 136 atmospheric detonations caused by meteors releasing blasts carrying an equivalent energy of nearly 1,000 tons of TNT - similar to small atomic bombs, but without the radiation. Because many were missed, the actual rates could be 10 times higher. If the radius of Earth is 6,378 km, A) what is the rate of these deadly impacts on Earth in terms of impacts per km^2 per year? B) Assuming that the impact rates are the same for Earth and the Moon, suppose a lunar colony has an area of 10 km^2. How many years would they have to wait between meteor impacts?

Problem 2 - Between 2005-2007, NASA astronomers counted 100 flashes of light from meteorites striking the lunar surface - each equivalent to as much as 100 pounds of TNT. If the surveyed area equaled 1/4 of the surface area of the Moon, and the lunar radius is 1,737 km, A) What is the arrival rate of these meteorites in meteorites per km^2 per year? B) If a lunar colony has an area of 10 km^2, how long on average would it be between impacts?

Problem 3 - According to H.J. Melosh (1981) meteoroids as small as 1-millimeter impact a body with a 100-km radius about once every 2 seconds. A) What is the impact rate in units of impacts per m^2 per hour? B) If an astronaut spent a cumulative 1000 hours moon walking and had a spacesuit surface area of 10 m^2, how many of these deadly impacts would he receive? C) How would you interpret your answer to B)?

Answer Key

Problem 1 - A) The surface area of Earth is $4 \pi (6378)^2 = 5.1 \times 10^8$ km^2. The rate is R = 136 x 10 impacts / 20 years / 5.1×10^8 km^2 = 1.3×10^{-7} impacts/km^2/year.

B) The number of impacts/year would be 1.3×10^{-7} impacts/km^2/year x 10 km^2 = 1.3×10^{-6} impacts/year . The time between impacts would be $1/1.3 \times 10^{-6}$ = 769,000 years!

Problem 2 - A) The total surface area of the Moon is $4 \pi (1737)^2 = 3.8 \times 10^7$ km^2. Only 1/4 of this is surveyed so the area is 9.5×10^5 km^2. Since 100 were spotted in 2 years, the arrival rate is R = 100 impacts/2 years/ 9.5×10^5 km^2 = 5.3×10^{-5} impacts/km^2/year.

B) The rate for this area is 10 km^2 x 5.3×10^{-5} impacts/km^2/year = 5.3×10^{-4} impacts/year, so the time between impacts is about 1/ 5.3×10^{-4} = 1,900 years

Problem 3 - A) A sphere 100-km in radius has a surface area of $4 \pi (100{,}000)^2 = 1.3 \times 10^{11}$ m^2. The impacts arrive every 2 seconds on average, which is 2/3600 = 5.6×10^{-4} hours. The rate is, therefore, R = 1 impacts / (1.3×10^{11} m^2 X 5.6×10^{-4} hours) = 1.4×10^{-8} impacts/m^2/hour.

B) The number of impacts would be 1.4×10^{-8} impacts/m^2/hour x 10 m^2 x 1000 hours = 1.4×10^{-5} impacts.

C) Because the number of impacts is vastly less than 1 (a certainty), he should not worry about such deadly impacts unless he had reason to suspect that the scientists miscalculated the impact rates for meteorites this small. Another way to look at this low number is to turn it around and say that the astronaut would have to take 1/ 1.4×10^{-5} about 71,000 such 1000-hour moon walks in order for one impact to occur. Alternately, the time between such events is 71,000 x 1000 hours = 71 million years!

The Earth and Moon to Scale

We have all seen drawings or sketches in books that show the earth and moon together in the same view, but in reality they are really very different in size, and are much farther apart than you might think.

By creating properly scaled drawings, you will get a better idea of what their sizes are really like! All you will need is a compass, a metric ruler, and a calculator.

The photo above was taken by the Voyager 1 spacecraft on September 18, 1977 at a distance of 7 million miles from Earth, and it has not been edited in any way. Are their diameters to scale? Their distance from each other? Even actual images can be distorted because of perspective and distance effects.

Problem 1 - The radius of the Moon is 1,737 kilometers, and the radius of Earth is 6,378 kilometers. What is the ratio of Earth's radius to the Moon's?

Problem 2 - To the nearest whole number, about how many times bigger than the Moon is Earth?

Problem 3 - With your ruler and compass, draw two circles that represent this size difference, and use a radius of 1 centimeter for the moon disk. Inside the circles, label them 'Earth' and 'Moon'.

Problem 4 - The distance between the center of Earth and the Moon is 384,000 kilometers. To the nearest integer, how many times the radius of Earth is the distance to the Moon?

Problem 5 - Cut out the circles for Earth and the Moon from Problem 3. Using the radius of your circle for Earth as a guide, how far apart, in centimeters, would you have to hold the two cut-outs to make a scale model of the Earth-Moon system that accurately shows the sizes of the two bodies and their distance?

Problem 6 - Look through books in your library, or use GOOGLE to do an image search. Do any of the illustrations show the Earth-Moon system in its correct scale? Why do you think artists draw the Earth-Moon system the way that they do?

Answer Key

Problem 1 - 6378 / 1737 = 3.7.

Problem 2 - 3.7 is closest to 4.0, so Earth is about 4 times bigger than the Moon in size.

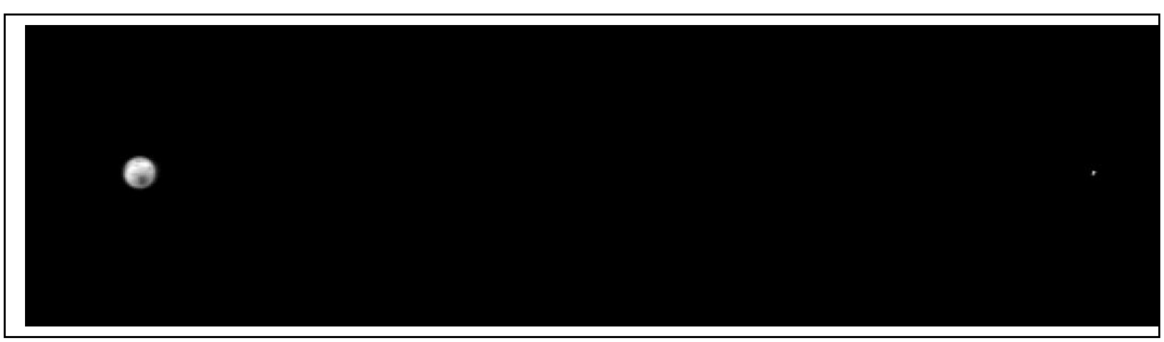
Problem 3 - Draw the disks on a separate paper, but make sure that the Moon has a 1 cm radius and Earth has a 4 cm radius.

Problem 4 - 384,000 / 6378 = 60.2 which is 60 times Earth's radius.

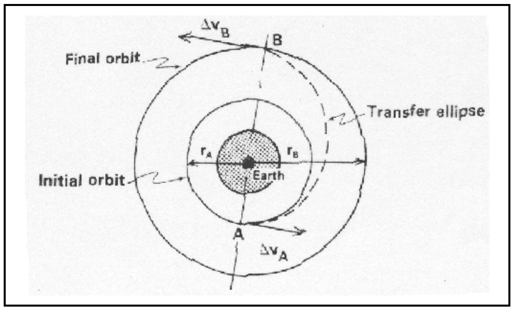
Problem 5 - If the radius of the Moon disk was 1 centimeter, the Earth disk would be 4 centimeters in radius. The distance to the Moon would be 60 times this distance, or 4 cm x 60 = 240 centimeters (2.4 meters).

Problem 6 - Very few. Artists try to show a vast 3-dimensional image in a flat perspective drawing that is only a few inches across on the printed page. To draw the Earth-Moon system in the proper perspective scale, the Moon would be a small dot. Also, illustrations that show the phases of the Moon, or eclipses, are also badly out of scale most of the time, because you can't show the phases clearly if the Moon is only the size of a small dot in the illustration. There are other purely artistic reasons too!

Note: The image below was taken by the Mars Odyssey spacecraft soon after launch in April 2001 when Earth and Moon were at their maximum separation. The image is not edited, and shows the disks at their true separations. From the diameter of Earth (12800 km) and its measured diameter in millimeters (4.5 mm) the scale of the image below is 12800/4.5 = 2840 km/mm. The separation from the center of Earth to the Moon 'dot' is 126.5 mm or 126.5 mm x 2840 km/mm = 359,000 km, which is close to its distance according to the US Almanac for April 2001 (…. Km). However, although Earth is 3.7x bigger than the Moon, it is clear that the lunar dot, which measures just under 0.5 mm) is much smaller than it should be (diameter = 3,474 km or 1.2 mm). **Compared to the Earth-Moon distance, how far from the Moon was the Mars Orbiter in order to see the Moon with this disk diameter? Answer: About 359,000 km x 1.2/0.5 = 860,000 km.**



Fly me to the Moon!



If spacecraft had rockets that could make them travel at any speed, we could fly to the Moon from Earth in a straight line, and make the trip in a few minutes. In the Real World, we can't do that even with the most powerful rockets we have. Instead, we have to obey Newton's Laws of Motion and take more leasurly, round-about routes!

To see how this works, you need a compass, metric ruler, a large piece of paper, a string, a thumbtack, and a pencil.

Step 1 - With your compass, draw a circle 1/2-centimeter in radius. Label the inside of this 'Earth'. **Step 2** - Draw a second circle centered on Earth with a radius of 1 centimeter. Label this 'Earth Orbit'. **Step 3** - Using the string and thumbtack, draw a second circle with a radius of 30 centimeters. Label this 'Orbit of Moon'. **Step 4** - Draw a line connecting the center of earth and a point on the lunar orbit. Label the lunar orbit Point B. **Step 5** - Extend the line so that it intersects a point on the Earth orbit circle in the opposite direction from Earth's center. There should be two intersection points. The first will be between Earth and the lunar orbit. Label this Point C. The second will be behind Earth. Label this Point A. **Step 6** - As carefully as you can, draw a free-hand ellipse with one focus centered on Earth that arcs between Point A and Point B. This is called the major axis of the ellipse. See the above figure for comparison.

What you have drawn is a simple rocket trajectory, called a Hohmann Transfer orbit, that connects a spacecraft orbiting Earth, with a point on the lunar orbit path. If you had unlimited rocket energy, you could travel the path from Point C to Point B in a few hours or less. If you had less energy, you would need to take a path that looks more like the one from Point A to Point B and is slower, so it takes more time. Even less energy would involve a spiral path that connects Point A and Point B but may loop one or more times around Earth as it makes its way to lunar orbit. Can you draw such a path?

Problem 1 - If there were no gravity, spacecraft could just travel from place to place in a straight line at their highest speeds, like the Enterprise in *Star Trek*. If the distance to the Moon is 380,000 kilometers, and the top speed of the Space Shuttle is 10 kilometers/sec, how many hours would the Shuttle take to reach the Moon?

Astrodynamicists are the experts that calculate orbits for spacecraft. One of the most important factors is the total speed change, called the delta-V, to get from one orbit to another. For a rocket to get into Earth orbit requires a delta-V of 8600 m/sec. To go from Earth orbit to the Moon takes an additional delta-V of 4100 meters/sec.

Problem 2 - To enter a Lunar Transfer Orbit, a spacecraft has enough fuel to make a total speed change of 3500 m/sec. If it needs to make a speed change of 2000 m/s in the horizontal direction, and 3000 m/sec in the vertical direction to enter the correct orbit, is there enough fuel to reach the Moon in this way? [Hint, use the Pythagorean Theorem]

Answer Key

Problem 1 - If there were no gravity, spacecraft could just travel from place to place in a straight line at their highest speeds, like the Enterprise in *Star Trek*. If the distance to the Moon is 380,000 kilometers, and the top speed of the Space Shuttle is 10 kilometers/sec, how many hours would the Shuttle take to reach the Moon?

Answer - Distance = speed x time, so 380,000 km = 10 km/s x time. Solving for Time you get 38,000 seconds. Since there are 60 minute/hour x 60 seconds/minute = 3600 seconds/hour, 38,000 / 3600 = 10.6 hours.

Problem 2 - To enter a Lunar Transfer Orbit, a spacecraft has enough fuel to make a total speed change of 3500 m/sec. If it needs to make a speed change of 2000 m/sec in the horizontal direction, and 3000 m/sec in the vertical direction to enter the correct orbit, is there enough fuel to reach the moon in this way? [Hint, use the Pythagorean Theorem, or solve graphically]

Answer:

Method 1: From the lengths of the horizontal and vertical speeds, we want to find the length of the hypotenuse of a right triangle. Using the Pythagorean Theorem

$$\text{total speed} = (2000^2 + 3000^2)^{1/2} = 3605 \text{ m/sec}$$

This is the total change of speed that is required, but there is only enough fuel for 3500 m/sec so the spacecraft cannot enter the Transfer Orbit.

Method 2: Graphically, draw a right triangle to the proper scale, for example, 1 centimeter = 1000 m/sec. Then the two sides of the triangle have lengths of 2 cm and 3 cm. Measure the length of the hypotenuse to get 3.6 cm, then convert this to a speed by multiplying by 1000 m/sec to get the answer.

Extracting Oxygen from Moon Rocks

About 85% of the mass of a rocket is taken up by oxygen for the fuel, and for astronaut life support. Thanks to the Apollo Program, we know that as much as 45% of the mass of lunar soil compounds consists of oxygen. The first job for lunar colonists will be to 'crack' lunar rock compounds to mine oxygen.

NASA has promised $250,000 for the first team capable of pulling breathable oxygen from mock moon dirt; the latest award in the space agency's Centennial Challenges program.

Lunar soil is rich in oxides of silicon, calcium and iron. In fact, 43% of the mass of lunar soil is oxygen. One of the most common lunar minerals is *ilmenite*, a mixture of iron, titanium, and oxygen. To separate *ilmenite* into its primary constituents, we add hydrogen and heat the mixture. This hydrogen reduction reaction is given by the 'molar' equation:

$$FeTiO_3 + H_2 \;\text{---}> \; Fe + TiO_2 + H_2O$$

__A Bit Of Chemistry__ - This equation is read from left to right as follows: One mole of *ilmenite* is combined with one mole of molecular hydrogen gas to produce one mole of free iron, one mole of titanium dioxide, and one mole of water. Note that the three atoms of oxygen on the left side (O_3) is 'balanced' by the three atoms of oxygen found on the right side (two in TiO_2 and one in H_2O). **One 'mole' equals 6.02 x 10^{23} molecules**.

The 'molar mass' of a molecule is the mass that the molecule has if there are 1 mole of them present. The masses of each atom that comprise the molecules are added up to get the molar mass of the molecule. Here's how you do this:

For H_2O, there are two atoms of hydrogen and one atom of oxygen. The atomic mass of hydrogen is 1.0 AMU and oxygen is 16.0 AMU, so the molar mass of H_2O is 2 (1.0) + 16.0 = 18.0 AMU. **One mole of water molecules will equal 18 grams of water by mass.**

Problem 1 -The atomic masses of the atoms in the *ilmenite* reduction equation are Fe = 55.8 and Ti = 47.9. A) What is the molar mass of ilmenite? B) What is the molar mass of molecular hydrogen gas? C) What is the molar mass of free iron? D) What is the molar mass of titanium dioxide? E) Is mass conserved in this reaction?

Problem 2 - If 1 kilogram of ilmenite was 'cracked' how many grams of water would be produced?

Inquiry Question - If 1 kilogram of ilmenite was 'cracked' how many grams of molecular oxygen would be produced if the water molecules were split by electrolysis into

$$2\,H_2O \;\text{--}> 2H_2 + O_2?$$

Answer Key

Problem 1 -

A) What is the molar mass of ilmenite? 1(55.8) + 1 (47.9) + 3(16.0) = 151.7 grams/mole
B) What is the molar mass of molecular hydrogen gas? 2(1.0) = 2.0 grams/mole
C) What is the molar mass of free iron? 1(55.8) = 55.8 grams/mole
D) What is the molar mass of titanium dioxide? 1(47.9) + 2(16.0) = 79.9 grams/mole
E) Is mass conserved in this reaction? Yes. There is one mole for each item on each side, so we just add the molar masses for each constituent. The left side has 151.7 + 2.0 = 153.7 grams and the right side has 55.8 + 79.9 + 18.0 = 153.7 grams so the mass balances on each side.

Problem 2 -

Step 1 - The reaction equation is balanced in terms of one mole of ilmenite (1.0 x $FeTiO_3$) yielding one mole of water (1.0 x H_2O) . The molar mass of ilmenite is 151.7 grams which is the same as 0.1517 kilograms, so we just need to figure out how many moles is needed to make one kilogram.

Step 2 - This will be 1000 grams/151.7 grams = 6.6 moles. Because our new reaction is that we start with 6.6 x $FeTiO_3$ that means that for the reaction to remain balanced, we need to produce 6.6 x H_2O, or in other words, 6.6 moles of water.

Step 3 - Because the molar mass of water is 18.0 grams/mole, the total mass of water produced will be 6.6 x 18.0 = 119 grams of water.

Inquiry Question - The reaction is: 2 H_2O -- > $2H_2$ + O_2

This means that for every 2 moles of water, we will get one mole of O_2. The ratio is 2 to 1. From the answer to Problem 2 ,we began with 6.59 moles of water not 2.0 moles. That means we will produce 6.6/2 = 3.3 moles of water. Since 1 molecule of oxygen has a molar mass of 2(16) = 32 grams/mole, the total mass of molecular oxygen will be 3.3 moles x 32 grams/mole = 106 grams. So, 1 kilogram of ilmenite will eventually yield 106 grams of breathable oxygen.

Additional Math Resources about the Moon

There are many web-related resources that cover the moon from a mathematical perspective. Here are just a few…

Lunar Math Lander – An interactive game in which you try to land a spacecraft by answering math questions without crashing the spacecraft on the moon.
http://www.kidsnumbers.com/division-moon-math.php

NASA-QUEST: Moon Math Guide – Cratering! – A complete hands-on guide to crater mathematics, and calculating irregular areas.
http://lcross.arc.nasa.gov/docs/MM_Suppl_Guide_v1.pdf

Moon Math Challenge Guide (ESA/NASA) – An inquiry-based guide to a variety of moon projects for grades 5-8 that involve students asking questions based on measurements thay can make.
http://esc.nasa.gov/documents/MoonMathChallengeGuide.pdf

PUMAS – Can an Astronaut on Mars Distinguish Earth from the Moon? Could the unaided eye of an observer on Mars tell apart the Earth and its moon, at their greatest separation?
https://pumas.gsfc.nasa.gov/examples/index.php?id=7

PUMAS – The Cause of the Phases of the Moon –This activity is frequently used to show the causes of the lunar phases. Generally a teacher tells students how to position the ball to show a full moon, a crescent moon, etc.
https://pumas.gsfc.nasa.gov/examples/index.php?id=83

PUMAS – The Moon Orbits the Sun? –Their observations, or what is “common knowledge”, lead them to believe the Moon does loops around the Earth. But is this true? A comparison of the gravitational forces of the Sun and Earth on the Moon hints at the answer to this question and a simple demonstration refutes the loop-view.
https://pumas.gsfc.nasa.gov/examples/index.php?id=86

The NASA Lunar Ephemeris – Gives distance to the moon for every day from the years 1995-2006. Can be used to explore lunar orbit shape, phases.
http://eclipse.gsfc.nasa.gov/TYPE/ephemeris.html

Interactive Lunar Ephemeris Calculator – calculates many lunar parameters for any day/year that you enter.
http://www.lunar-occultations.com/rlo/ephemeris.htm

Lunar Cratering Rates – How scientists calculate the rate of cratering on the lunar surface. Includes a graph showing crater size versus frequency.
http://www.psi.edu/projects/mgs/cratering.html

A note from the Author,

It is hard to believe that it has been over 36 years since Apollo-17 left the moon, and the last humans walked on its surface. It is difficult to look back at the intervening years and not dwell upon the opportunities that were lost in setting up the first human colonies there. Back then, there was a huge public outcry against the perceived wastefulness of sending humans to the moon, when so many humans remained to be fed here on Earth.

Now, of course, the human population has nearly doubled, (3.8 billion in 1972 to 6.7 billion). Despite the trillions of dollars that have been spent to relieve poverty, it is still with us. The 0.7% of our federal budget that we are allowed to spend on space exploration ($17 billion in 2007) is completely overshadowed by other huge purchases that we seldom complain about (for instance, bottled water $12 billion and pet food $17 billion), and the nearly $1 trillion now deeded to 'bail-out' our economy.

This math guide celebrates the impending return to the moon by NASA with the launch of the Lunar Reconnaissance Orbiter (LRO). It is the first mission in NASA's plan to return to the moon, and then to travel to Mars and beyond. LRO will launch sometime in 2009. Its main goals are to search for safe landing sites, locate potential resources for creating fuels and food such as water, and measure the radiation environment.

I hope you will enjoy this sample of problems, and help your students capture some of the excitement of exploring the universe through mathematics!

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